

TAKING INSTRUMENT ERRORS INTO ACCOUNT IN SOLVING
INVERSE HEAT-CONDUCTION PROBLEMS WITH SPECIALIZED
ANALOG COMPUTERS

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UDC 536.24

The article suggests a method of taking into account the instrument errors arising in the solution of inverse heat-conduction problems on specialized analog computers by the method with self-tuned models.

An important problem of thermal identification is the determination of thermophysical characteristics (TPC) of materials in a wide temperature range.

Experimental investigations of the TPC of heat-insulating coatings are carried out under other than real conditions for a number of reasons (liberation of gaseous products, removal of coking products, heating rates). Another important case, where it is difficult to use thermophysical measurements based on classical approaches, is the investigation of the thermophysics of semitransparent materials.

These incongruities can be resolved by solving inverse heat-conduction problems (IHCP) for obtaining the TPC.

The object of the present work is to investigate instrument errors in solving IHCP on electrical models consisting of resistors and capacitors (RC models). The problem consists in determining the TPC of the material when full-scale thermal experiments are carried out. Special attention must be given to evaluating and taking into account the errors in solving IHCP on RC models.

At present examples are known of the successful application of analog simulation devices for solving IHCP. One of the promising methods of solving IHCP on analog computers is the method with self-tuned models. For that it is necessary to have an analog model for the identified object and also the possibility of measuring the input effect $U(\tau)$ and the response of the object $Y(\tau)$. The adaptation device changes the parameters of the model for attaining the specified discrepancy Δ between the responses of the object and of the model (Fig. 1). Problems of using self-tuned models in the form of RC-grids are not given much attention nowadays.

When the method with self-tuned models is used, the errors occurring in the solution of the problem may be divided into three groups: methodical, instrument errors, and errors of processing the results.

The sources of methodical errors are the following: discretization of the space in modeling on RC-grids, incorrect selection of the point of measuring the response of the object, imperfect regularity of control arranged in the adaptation device. The errors occurring in the discretization of the space are determined by the pitch of the grid. It was experimentally proved [1]: for the error of discretization to tend to zero, it suffices in many cases to ensure the condition $\Delta x/\delta \leq 0.1$. However, with increasing number of cells of the model (to make $\Delta x/\delta$ smaller) the instrument errors increase. As a compromise solution it is suggested to use models consisting of 8-10 RC-cells [1].

The magnitude of the methodical errors of the regularity of control arranged in the adaptation device is determined by the qualitative indicators of the regulating process: the time of regulation, the number of oscillations within the time of regulation, and the steady error.

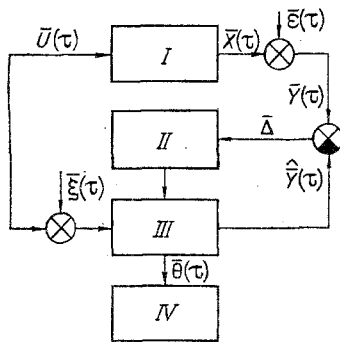


Fig. 1

Fig. 1. Structural diagram of identifying objects with the aid of a self-tuned model: I) object; II) adaptation device; III) model; IV) measuring system.

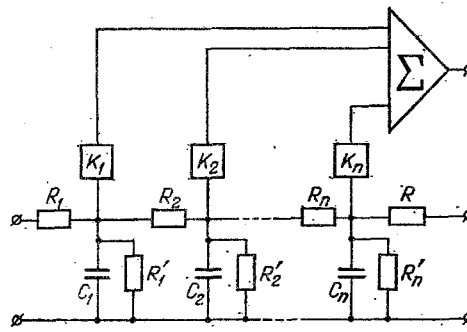


Fig. 2

Fig. 2. Functional diagram of a unidimensional RC-grid with a device for automatically taking into account the errors due to leakages.

In solving IHCP, the accuracy of determining the initial information (for the adaptation device it is the temperature of the identified object measured at the distance x_1 from the heat-exchange surface) is of first-rate importance. When a thermal process is proceeding, it is always possible to distinguish two regions in the direction of the x -coordinate. When the temperature sensor is situated in the first region, it is possible in principle to solve the IHCP. The position of the boundary between the regions is determined by the thermal diffusivity of the material of the body, the nature of the thermal effect, and the sensitivity of the measuring system. Therefore the sensor has to be situated in the zone of reliable temperature measurement.

Very important is the position of the sensor within the boundaries of the zone of reliable temperature measurement. The time during which the experiment is carried out consists of two intervals: the length of the passive experiment and the length of the active experiment. When the duration of the experiment is limited, there exists a minimally permissible duration of the active experiment (determined by the regularity of control) with which a solution of the IHCP can be obtained. To shorten the time of the passive experiment, the sensor should be placed as close as possible to the heat-exchange surface.

Among the instrument errors are errors in mounting the thermocouples, of the introduction of boundary conditions, of the adaptation device and of the measuring system, and also specific errors of the method under consideration introduced by the adaptation device. The unidimensional process in an electric model with and without sinks is represented in differential form [1]:

$$\frac{\partial}{\partial x} \left(\frac{1}{R} \frac{\partial U}{\partial x} \right) = C \frac{\partial U}{\partial \tau}, \quad (1)$$

$$\frac{\partial}{\partial x} \left(\frac{1}{R} \frac{\partial U}{\partial x} \right) - \frac{U}{R_{si}} = C \frac{\partial U}{\partial \tau}. \quad (2)$$

Self-tuning of the RC-grid is effected by changing the resistances or the capacities [1, 2]. Analog devices, intended for the tuning of the grid, inevitably distort the modeling process. For analyzing errors, such devices can often be represented in the form of an active linear leakage resistance. Thus, instead of the desirable Eq. (1), the grid will in fact be used to solve the equation with sinks (2). An obvious way of reducing the errors due to leakages is the reduction of the number of cells of the grid and the use of discharge repeaters with high input resistance. However, such a method is not universal.

The difficulty of taking errors due to leakages into account consists in the fact that the magnitude of the error is a function of time and of the parameters of the model.

For a unidimensional RC-grid, with leakages occurring, the voltage equation in operator form may be represented in the form

$$U_i(p) = \left[(U_{C1}T_{1,p} + U_{C2}T_{1,2,p} + \dots + U_{Cn}T_{1,n,p}) + \left(\frac{R_1}{R_1} U_{C1} + \frac{R_1}{R_2} U_{C2} + \dots + \frac{R_1}{R_n} U_{Cn} \right) \right] + \quad (3)$$

$$+ \left[(U_{C2}T_{2,p} + \dots + U_{Cn}T_{2,n,p}) + \left(\frac{R_2}{R_2} U_{C2} + \dots + \frac{R_2}{R_n} U_{Cn} \right) \right] + \dots + \left[U_{Cn}T_{n,p} + \frac{R_n}{R_n} U_{Cn} \right] + U_{Cn}.$$

It can be seen from expression (3) that for compensating the errors caused by leakages, the following correction has to be introduced in measuring voltages at the nodes of the grid:

$$\Delta U_i = \sum_{m=1}^n \frac{R_i}{R_m} U_m. \quad (4)$$

In the case of an analog adaptation device, the correction may be introduced automatically with the aid of adders (Fig. 2).

To evaluate the efficiency of the suggested method, we solved test problems on a specialized RC-grid. We modeled a linear nonsteady process of heat transfer with boundary conditions of the third kind. Modeling was carried out with two models: the first one was an ordinary one [1], and into the second we introduced leakages imitating the analog adaptation devices. The solution of one of the problems is represented in Fig. 3.

The experiments showed that the relative errors of determining temperatures, caused by leakages, may attain impermissible values (20% or more).

The experiments confirmed the efficiency of the suggested method; its application will make it possible to reduce instrument errors of units of specialized analog computers to 2-3%.

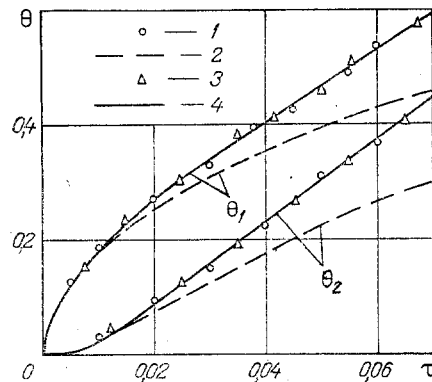


Fig. 3. Results of the solution of the test problem: 1) model without leakages; 2) model with leakages; 3) model taking leakages automatically into account; 4) exact solution. $\alpha = 1800 \text{ W/m}^2 \cdot \text{K}$; $\lambda = 45 \text{ W/m} \cdot \text{K}$; $\delta = 0.005 \text{ m}$; $a = 125 \cdot 10^{-7} \text{ m}^2/\text{sec}$; τ , sec; θ_1, θ_2) relative (dimensionless) temperature of the heated and cooled wall surfaces, respectively.

NOTATION

α , thermal diffusivity; λ , thermal conductivity; α , heat-transfer coefficient; θ , relative temperature; δ , wall thickness; τ , time; C , electrical capacity; R , electrical resistance; R' , leakage resistance; $K = R/R'$, coefficient; $T = RC$, time constant; n , number of nodal points of the grid; U , voltage; p , Laplace operator; x , coordinate; Δx , discretization step of the coordinate; $X(\tau)$, vector of state of the object; $Y(\tau)$, vector of observation of the object; $\bar{U}(\tau)$, control vector; $\Theta(\tau)$, vector of the unknown parameters; $\varepsilon(\tau)$, $\xi(\tau)$, vectors of measurement noise; $\hat{\cdot}$, estimate.

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AUTOMATIC SYSTEM OF IDENTIFYING THERMOPHYSICAL PARAMETERS OF OBJECTS ON THE BASIS OF SOLUTION OF INVERSE HEAT-CONDUCTION PROBLEMS

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UDC 536.24.083

The article examines an automated system of determining the parameters of heat exchange and the thermophysical characteristics of the investigated objects on the basis of the solution of inverse heat-conduction problems (IHCP).

Improving the quality and effectiveness of thermophysical research entails the necessity of devising and introducing modern means of automating such research. In accordance with the material of [1], automatic systems of scientific research (ASSR) are based on obtaining and using models of objects, phenomena, and processes. The methods and algorithms for processing the experimental data, based on the solution of the corresponding IHCP, are at present one of the most universal methods of experimental thermophysics, and they are the natural basis of the mathematical provisions for the planned ASSR of thermal processes. Planning ASSR must take into account the special features, not only of the studied thermal processes and objects, but also of the algorithms for solving the IHCP connected with the solution of boundary problems described by differential equations in partial derivatives.

The selection of the structural schema and the organization of the work of ASSR, of the parameters of heat exchange and of the thermophysical characteristics of the investigated objects (carried out at the Institute of Technical Thermophysics, Academy of Sciences of the Ukrainian SSR) are based on the use of standard schemata of the control computer complexes (CCC) SM-3 and SM-4 described in [2]. The principal requirements that ASSR have to fulfill are the following: a) sufficiently rapid operation of the channel sensor-amplifier-commutator-ADC-IM which, in view of the programming provisions, has to amount to at least 500 Hz; this makes it possible to record reliably the readings of 20-30 thermocouples under the non-steady conditions of the highly forced process of heat exchange (thermal flux on the surface of the investigated object $\alpha_f > 10^5 \text{ W/m}^2$); b) sufficiently rapid operation (at least 20,000 operations per second) and capacity of the IM (at least 10 Kbytes) of the digital processor that is part of the ASSR. Figure 1 shows the structural schema of an ASSR that makes it possible to investigate in regimes of actively affecting the object. The complex is based on the CCC SM-4 embodying the processor SM-4P with a speed of 800,000 operations per second and an IM with a capacity of 56 Kbytes. The connection with the hardness is effected in the

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Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 45, No. 5, pp. 825-828, November, 1983.
Original article submitted February 2, 1983.